Science Supplementary Material
SM358 The Quantum World

The Open University

SM358

Glossary for Book 1

This is a self-contained glossary for Book 1. Numbers in parentheses are page references with B1 denoting Book 1 (*Wave mechanics*).

Italicized words are cross-references to other entries in this Glossary.

absolute temperature A temperature measured on the *absolute temperature scale*.

absolute temperature scale The *SI* scale of temperature measured in *kelvin* (K). On this scale, the lowest conceivable temperature, *absolute zero*, is $0\,\mathrm{K}$.

absolute zero The lowest conceivable temperature for any *system*. It is represented by the value $0\,\mathrm{K}$ on the *absolute temperature scale*, and corresponds to a temperature of $-273.15\,^\circ\mathrm{C}$ on the Celsius temperature scale. In *classical physics*, where temperature is a measure of molecular agitation, absolute zero corresponds to all *particles* being in a *state* of rest with a minimum mutual *potential energy*.

addition rule for probability (B1: 228) This mathematical rule states that the *probability* of obtaining one or other of a set of *mutually exclusive outcomes* in a single trial or experiment is the sum of their individual probabilities.

alpha decay (B1: 203) A form of *radioactive decay* in which the *nucleus* of an *atom* ejects an energetic *alpha particle*. As a result, the *atomic number* Z of the emitting nucleus is reduced by two, and its *mass number* A is reduced by four.

Alpha decay involves the *tunnelling* of alpha particles through a *Coulomb barrier*. See also the *Geiger–Nuttall relation*.

alpha particle (B1: 12) A type of composite *particle*, consisting of two *protons* and two *neutrons* bound together. An alpha particle is identical to the *nucleus* of a *helium atom* with *mass number* 4, and is therefore the same as a doubly-*ionized* helium-4 atom. The term is generally used in the context of alpha-particle emission from certain unstable nuclei in the *radioactive decay* process known as *alpha decay*.

amplitude (B1: 17, 125) The *magnitude* of the maximum deviation of an *oscillation* or *wave* from equilibrium. For a *sinusoidal* oscillation described by the function $x(t) = A\cos(\omega t + \phi)$, the positive constant A is the amplitude of the oscillation. For a sinusoidal wave described by the function $u(x,t) = A\cos(kx - \omega t + \phi)$, the positive constant A is the amplitude of the wave.

In *quantum physics*, the word 'amplitude' is sometimes used as a shorthand for *probability amplitude*; this should not be confused with the amplitude of an oscillation or wave.

angular frequency (B1: 10, 17, 126) The rate of change of the *phase* of an *oscillation* or *wave*. The angular frequency ω is given by

$$\omega = 2\pi f = \frac{2\pi}{T},$$

where f is the *frequency* and T is the *period* of the oscillation or wave. The *SI* unit of angular frequency is the inverse second, s^{-1}. (Some authors use $\mathrm{rad\,s}^{-1}$, but this clashes with the fact that product of angular frequency and time appears in functions such as $\sin(\omega t)$ and, in this context, ωt must be a pure number to ensure that an expansion of the sine function gives a power series in which different terms have the same units.)

arbitrary constant (B1: 218) A constant that appears in the *general solution* of a *differential equation* but does not appear in the differential equation itself. If the *order of a differential equation* is n, the general solution of the equation contains n arbitrary constants. The arbitrary constants serve to distinguish one *particular solution* from another.

Argand diagram (B1: 212) Another term for the *complex plane*.

argument (B1: 213) (i) A term for the *phase* of a *complex number*.

(ii) Another term for an expression within a function; for example, the argument of $\sin(kx)$ is kx.

atom The smallest electrically-neutral sample of an *element* that retains the fundamental chemical and physical identity of that element. An atom consists of a positively-charged *nucleus* surrounded by a cloud of negatively-charged *electrons*. Most of the mass of an atom is contained in its nucleus, but atomic sizes are generally determined by the distribution of electrons. Atomic radii vary from about 5×10^{-11} m to 3×10^{-10} m.

atomic number (B1: 12) The number of *protons* in the *nucleus* of a particular type of *atom*, and hence the number of *electrons* in the neutral atom. An *element* is identified by its atomic number Z, which determines its chemical properties. The atomic number is usually denoted by the symbol Z; *hydrogen*, *helium* and lithium atoms have $Z = 1$, 2 and 3 respectively.

attenuation coefficient (B1: 199) A *real* quantity that determines the rate of exponential decrease of some other quantity with increasing distance. An example is the quantity α that appears in the expression $Ce^{-\alpha|x|}$, which is the form of a *bound state energy eigenfunction* for a *particle* in one of the *classically-forbidden regions* outside a one-dimensional *finite square well*. The *SI* unit of an attenuation coefficient is m^{-1}.

auxiliary equation (B1: 219) An algebraic equation obtained when a trial solution containing undetermined parameters is substituted into a *differential equation*. The auxiliary equation determines possible values of these parameters.

average value (B1: 228) If a quantity A is measured N times in a given situation, and the result A_i is obtained on N_i occasions, the average value of A over the set of measurements is defined to be

$$\overline{A} = \frac{1}{N}\sum_{i=1}^{N} N_i A_i = \sum_{i=1}^{N} f_i A_i,$$

where the sum is over all the outcomes and $f_i = N_i/N$ is the *relative frequency* of outcome i.

The measured average value $\overline{A}$ is expected to approach the theoretical *expectation value* $\langle A \rangle$ as the number of measurements becomes very large. Also called the *mean value*.

barn (B1: 196) A unit of area widely used to measure *total cross-sections* in *elementary particle* and nuclear physics. 1 barn $= 10^{-28}\,\mathrm{m}^2$.

barrier penetration (B1: 88, 133) The quantum phenomenon whereby *particles* may be detected in a *classically-forbidden region*. Compare with *tunnelling*.

beam intensity (B1: 183) For one-dimensional *scattering* and *tunnelling* of a beam of *particles*, the beam intensity is the number of beam particles that pass a given point per unit time. The beam intensity can be identified with the *magnitude* of the corresponding *probability current*. Also called *intensity of a beam*. Compare with *flux* for three-dimensional scattering.

beam splitter (B1: 15) A device that splits a beam of *photons* (or other *particles*) into two or more distinct sub-beams. When a single photon passes through a beam splitter, it emerges in a *linear superposition* of the *states* associated with the two output sub-beams, although we cannot say which way the photon went until it is actually detected.

A *half-silvered mirror* can be used as a beam splitter.

Born's rule (B1: 24) For a single *particle* in one dimension, in a *state* described by the *wave function* $\Psi(x,t)$, Born's rule tells us that, at time t, the *probability* of finding the particle in a small interval δx, centred on position x, is

$$\text{probability} = |\Psi(x,t)|^2\,\delta x.$$

For a single particle in three dimensions, in a state described by the wave function $\Psi(\mathbf{r},t)$, the probability, at time t, of finding the particle in a small volume element δV, centred on position $\mathbf{r}$, is

$$\text{probability} = |\Psi(\mathbf{r},t)|^2\,\delta V.$$

Born's rule for momentum (B1: 170) For a one-dimensional *free-particle wave packet*,

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} A(k)\,e^{i(kx-E_k t/\hbar)}\,dk,$$

Born's rule for momentum states that the *probability* of finding the *momentum* to lie in a small interval $\hbar\,\delta k$, centred on $\hbar k$, is $|A(k)|^2\,\delta k$, where $A(k)$ is called the *momentum amplitude* function.

The *magnitude* of the momentum amplitude function is independent of time for a free particle, but this is not true for a *particle* subject to forces. More generally, Born's rule for momentum states that, at time t, the probability of finding the momentum to lie in a small interval $\hbar\,\delta k$, centred on $\hbar k$, is $|A(k,t)|^2\,\delta k$, where $A(k,t)$ is the momentum amplitude function at time t, given by the *Fourier transform* of the *wave function*:

$$A(k,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \Psi(x,t)e^{-ikx}\,dx.$$

Compare with *Born's rule*.

bound state (B1: 91) A *state* in which one part of a *system* is always found in close proximity to another part of the same system. The *wave functions* describing bound states are always *normalizable*, and so tend to zero as we move away from the system. The *energy eigenvalues* of bound states are discrete. Contrast with a state in the *continuum*.

bound system A *system* that is in a *bound state*.

boundary conditions (B1: 221) Conditions that give extra information about the solutions of a *differential equation*. Such information may be sufficient to determine all the *arbitrary constants* in the *general solution* of a differential equation, and so select a *particular solution*, or it may determine acceptable *eigenvalues* and *eigenfunctions* that satisfy an *eigenvalue equation*.

Cartesian coordinate system A set of three mutually perpendicular axes pointing outwards from a single origin. The axes are called the x-axis, the y-axis and the z-axis. Such a coordinate system is usually chosen to be right-handed so that, if the fingers of the right hand initially point in the x-direction, and are then bent to point in the y-direction, the outstretched right thumb points in the z-direction.

Cartesian coordinates Coordinates x, y and z that represent the position of a point relative to a given *Cartesian coordinate system*.

Cartesian form (B1: 213) A *complex number* z is said to be expressed in Cartesian form if it is written as $z = x + \mathrm{i}y$, where x and y are *real numbers*.

centre-of-mass frame (B1: 127) A *frame of reference* whose origin permanently coincides with the centre of mass of a given *system* of *particles*.

charge Often used as an abbreviation for *electric charge*.

classical limit (B1: 160) Limiting conditions under which the predictions of *quantum mechanics* approach those of *classical mechanics*. The classical limit generally involves objects that are much larger than *atoms*, and forces that vary slowly over the width of the *wave packet* describing the object. See also the *correspondence principle* and *Ehrenfest's theorem*.

classical mechanics A theory of the behaviour of *systems* of *particles* based on Newton's laws. Fundamental equations in classical mechanics can be expressed in terms of the *Hamiltonian function*. Also called Newtonian mechanics.

classical physics (B1: 7) A term given to branches of physics that do not rely on quantum ideas. Classical physics embraces *classical mechanics* and subjects such as classical electromagnetism, fluid mechanics and thermodynamics. Most physicists regard special and general relativity as belonging to classical physics, which implies that the major revolution of twentieth-century physics was *quantum physics*, not relativity.

classically-forbidden region (B1: 66) A region of space from which a *particle* is excluded in *classical mechanics*. The exclusion arises because the fixed total *energy* of the particle is less than the value of the *potential energy function* throughout the classically-forbidden region. A classically-forbidden region may be accessible in *quantum mechanics* through the phenomenon of *barrier penetration*.

coefficient rule (B1: 105) This rule states that, if the *wave function* $\Psi(x, t)$ of a *system* is expressed as a discrete *linear combination* of *normalized energy eigenfunctions*:

$$\Psi(x, t) = c_1(t)\,\psi_1(x) + c_2(t)\,\psi_2(x) + \dots\,,$$

then the *probability* of obtaining the ith *energy eigenvalue* E_i is

$$p_i = |c_i(t)|^2\,,$$

where $c_i(t)$ is the coefficient of the ith energy eigenfunction in the wave function at the instant of *measurement*.

This rule can be extended to any *observable*, A, with a discrete set of values provided that the wave function is expanded as a linear combination of *eigenfunctions* of the corresponding quantum-mechanical *operator* $\widehat{\mathrm{A}}$.

coherent waves Two *waves* are said to be coherent with one another if knowledge of the *phase* of one wave at a particular position and time enables the phase of the other wave to be predicted at some other position and time.

collapse of the wave function (B1: 27, 97) The abrupt and unpredictable change in a *wave function* that arises during an act of *measurement*. This collapse cannot be described by *Schrödinger's equation*. An example is the collapse that occurs when the position of a *particle* is measured by a *Geiger counter*; immediately after the Geiger counter has clicked, the wave function describing the particle is localized in the vicinity of the Geiger counter.

commutation relation (B1: 138) An equation for the *commutator* of two given *operators*. For example, the *lowering* and *raising operators* of a *harmonic oscillator* obey the commutation relation

$$\left[\widehat{\mathrm{A}}, \widehat{\mathrm{A}}^\dagger\right] = 1.$$

commutator (B1: 138) The commutator of two *operators* $\widehat{\mathrm{A}}$ and $\widehat{\mathrm{B}}$ is itself an operator, denoted by the symbol $\left[\widehat{\mathrm{A}}, \widehat{\mathrm{B}}\right]$, and defined by

$$\left[\widehat{\mathrm{A}}, \widehat{\mathrm{B}}\right] = \widehat{\mathrm{A}}\widehat{\mathrm{B}} - \widehat{\mathrm{B}}\widehat{\mathrm{A}}.$$

See also *commuting operators*.

commute See *commuting operators*.

commuting operators Two *operators* $\widehat{\mathrm{A}}$ and $\widehat{\mathrm{B}}$ *commute* with each other if they have the same effect no matter which order they are applied. The result of operating on a function $f(x)$ with $\widehat{\mathrm{A}}$ followed by $\widehat{\mathrm{B}}$ is $\widehat{\mathrm{B}}\widehat{\mathrm{A}}f(x)$, while the result of operating on $f(x)$ with $\widehat{\mathrm{B}}$ followed by $\widehat{\mathrm{A}}$ is $\widehat{\mathrm{A}}\widehat{\mathrm{B}}f(x)$. For commuting operators, these two results are always the same, for all functions $f(x)$. We then write

$$\widehat{\mathrm{A}}\widehat{\mathrm{B}} = \widehat{\mathrm{B}}\widehat{\mathrm{A}}.$$

Equivalently,

$$[\widehat{\mathrm{A}}, \widehat{\mathrm{B}}] = \widehat{\mathrm{A}}\widehat{\mathrm{B}} - \widehat{\mathrm{B}}\widehat{\mathrm{A}} = 0,$$

so the *commutator* of two commuting operators is equal to zero.

complete set of functions (B1: 164) A discrete set of functions, $\phi_i(x)$ labelled by the index $i = 0, 1, 2, \ldots$, is said to be complete if it is possible to expand any reasonable function $f(x)$ in the form

$$f(x) = \sum_{i=0}^{\infty} c_i\, \phi_i(x),$$

where the coefficients c_i are constants (possibly *complex*, but independent of x). The *harmonic-oscillator energy eigenfunctions* are complete in this sense.

This definition is extended to a continuous set of functions $\phi_k(x)$, labelled by a continuous index k, by replacing the sum by an integral:

$$f(x) = \int_{-\infty}^{\infty} C(k)\, \phi_k(x)\, \mathrm{d}k,$$

where $C(k)$ is some complex-valued function. The *momentum eigenfunctions* are complete in this sense.

complete set of outcomes (B1: 228) A set of *mutually exclusive outcomes* for an experiment is said to be complete if every possible outcome of the experiment is a member of the set.

complex conjugate (B1: 212) For a given *complex number* z, the complex conjugate z^* is the complex number obtained by reversing the sign of i wherever it appears in z. So, given a complex number

$$z = x + \mathrm{i}y = r(\cos\theta + \mathrm{i}\sin\theta) = r\mathrm{e}^{\mathrm{i}\theta},$$

where x, y, r and θ are *real*, the complex conjugate of z is

$$z^* = x - \mathrm{i}y = r(\cos\theta - \mathrm{i}\sin\theta) = r\mathrm{e}^{-\mathrm{i}\theta}.$$

complex number (B1: 210) An entity that can be written in the form

$$z = x + \mathrm{i}y,$$

where x and y are *real numbers* and $\mathrm{i} = \sqrt{-1}$. Complex numbers obey the ordinary rules of algebra, with the extra rule that $\mathrm{i}^2 = -1$. See also *polar form*, *exponential form*, *argument*, *phase* and *complex conjugate*.

complex plane (B1: 212) A plane in which *complex numbers* in the *Cartesian form* $z = x + \mathrm{i}y$ are represented by points with *Cartesian coordinates* (x, y). *Real numbers* are represented by points along the (horizontal) x-axis, and *imaginary numbers* are represented by points along the (vertical) y-axis. Also called the *Argand diagram*.

conservation of energy (B1: 105) The principle that the total *energy* of any *isolated system* remains constant in time. In applying this principle, it is essential to include the energies of any *photons* that are absorbed or emitted.

In *quantum physics*, where a system may have an indefinite energy, conservation of energy is the principle that the *probability distribution* of energy, in any isolated system, remains constant in time. It then follows that the *expectation value* and the *uncertainty* of the energy remain constant in time.

constructive interference (B1: 28) The phenomenon in which two or more *waves* reinforce one another when they are superimposed at a given point. Contrast with *destructive interference*.

continuity boundary conditions (B1: 68) *Boundary conditions* that refer to the continuity of *energy eigenfunctions* and their derivatives. The energy eigenfunction $\psi(x)$ is always continuous. The first derivative $\mathrm{d}\psi/\mathrm{d}x$ is continuous in regions where the *potential energy function* is finite; it need not be continuous at points where the potential energy function becomes infinite. Note that *wave packets* composed of an infinite number of energy eigenfunctions are not subject to continuity boundary conditions.

continuous probability distribution (B1: 231) A *probability* distribution for a continuous *random* variable, specified by an appropriate *probability density function*.

continuum (B1: 12) (i) The name given to an infinite set of *energy levels* over which the *energy* varies continuously. The *state* of a *particle* in the continuum cannot be described by a *stationary-state wave function* of definite energy because such a function cannot be *normalized*; instead, it is described by a *wave packet* of indefinite energy.

(ii) In mathematics, any continuous set of values (as opposed to a discrete set of values) is said to form a continuum.

correspondence principle (B1: 135) The general principle that the predictions of *quantum mechanics* should approach those of *classical mechanics* in the limit of high *quantum numbers*. See also *classical limit*.

Coulomb barrier (B1: 204) The repulsive barrier, described by the *Coulomb potential energy function*

$$V(r) = \frac{Qq}{4\pi\varepsilon_0 r},$$

that is encountered by a *particle* of *charge* q when it is a distance r from a fixed point-like charge Q, where q and Q have the same sign. (If q and Q have opposite signs, the repulsive barrier becomes an attractive well.)

Coulomb force The *electrostatic force* between two charged *particles*, given by *Coulomb's law*.

Coulomb potential energy function A function specifying the mutual *potential energy* of two charged *particles* in a vacuum. This is given by

$$E_{\text{pot}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r},$$

where q_1 and q_2 are the *charges* of the particles, r is their separation and ε_0 is the *permittivity of free space*. By convention, the *zero of potential energy* is taken to correspond to infinite separation of the particles, so the Coulomb potential energy of oppositely-charged particles is negative.

Coulomb's law The physical law quantifying the *electrostatic force* between two stationary charged *particles*. This force acts along the line joining the particles; it is repulsive if the *charges* have the same sign and attractive if they have opposite signs. The force has *magnitude*

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2},$$

where q_1 and q_2 are the charges of the particles, r is the distance between them and ε_0 is the *permittivity of free space*.

cycle The shortest part of an *oscillation* or *wave* over which the motion repeats itself; a cycle endures for exactly one *period*.

de Broglie relationship (B1: 21) The relationship between the *de Broglie wavelength* λ_{dB} and the *magnitude* p of the *momentum* of a *free particle*:

$$\lambda_{\text{dB}} = \frac{h}{p},$$

where h is *Planck's constant*.

de Broglie wave A *wave* used in *quantum physics* to describe a *free particle*.

de Broglie wave function (B1: 27) A *wave function* of the form $A\mathrm{e}^{\mathrm{i}(kx-\omega t)}$ that describes a *stationary state* of a *free particle* in *quantum mechanics*. This is a *wave* of *amplitude* $|A|$, propagating in the x-direction with *wave number* k and *angular frequency* ω, corresponding to a free particle with *momentum* $p_x = \hbar k$ and *energy* $E = \hbar\omega$.

A de Broglie wave function does not describe a fully-realistic *quantum state* because it cannot be *normalized*, but realistic wave functions for free particles can be formed by taking continuous *linear combinations* of de Broglie wave functions (free-particle *wave packets*).

de Broglie wavelength (B1: 21) The *wavelength* of a *de Broglie wave*, given by the *de Broglie relationship*, $\lambda_{\text{dB}} = h/p$.

decay constant (B1: 13) A quantity, λ, with units of inverse time, characterizing the *radioactive decay* of a specific kind of *nucleus* (or the *particle decay* of a specific kind of *particle*). The *probability* of decay in a short time interval δt is $\lambda\,\delta t$, which is independent of time. Given a large sample of nuclei of the same kind, the average number of nuclei that remain undecayed at time t falls exponentially according to the *exponential law of radioactive decay*,

$$N(t) = N(0)\,\mathrm{e}^{-\lambda t}.$$

where $N(0)$ is the initial number of nuclei at time $t = 0$. See also *half-life*.

degeneracy (B1: 81) The occurrence of different *quantum states* with the same *energy*.

degenerate (B1: 81) Two *quantum states* are said to be degenerate with one another if they have the same *energy*. An *energy level* that corresponds to more than one quantum state is also said to be degenerate.

dependent variable In a function $f(x, y, \ldots, z)$, the dependent variable is f. Contrast with the *independent variables* $x, y, \ldots, z$.

destructive interference (B1: 28) The phenomenon in which two or more *waves* cancel one another when they are superimposed at a given point. Contrast with *constructive interference*.

deterministic (B1: 13) A term indicating that identical *initial conditions* lead to identical outcomes on all occasions. For example, *classical mechanics* is a deterministic theory. Contrast with *indeterministic*.

deuterium atom An *isotope* of *hydrogen* with *mass number* $A = 2$. A neutral deuterium atom has a single *electron* outside a deuterium *nucleus*, which consists of a *bound state* of a *proton* and a *neutron* (a *deuteron*).

deuteron (B1: 91) A *bound state* of a *proton* and a *neutron*. A deuteron is the *nucleus* of a *deuterium atom*.

diatomic molecule A *molecule* consisting of two *atoms* bound together.

differential cross-section (B1: 196) A quantity used to measure the rate per unit incident *flux* per unit solid angle, at which a given type of target scatters a given type of incident *particle* into a small cone of angles around a specified direction. The integral of the

differential cross-section over all directions is equal to the *total cross-section*. Compare with *reflection coefficient* in one dimension.

differential equation (B1: 217) An equation that involves derivatives of a function. *Ordinary differential equations* involve ordinary derivatives, while *partial differential equations* involve *partial derivatives*. However, the term 'differential equation' is often used as a shorthand for 'ordinary differential equation'. The process of solving the differential equation involves finding functions that satisfy the equation. We may be interested in finding *general solutions*, or finding *particular solutions* that satisfy the equation together with appropriate *boundary conditions* or *initial conditions*.

diffraction (B1: 16) The spreading of a *wave* that occurs when it passes through an aperture or around an obstacle. Diffraction is caused by the *interference* of waves taking different routes. Classical examples include the diffraction of water waves, sound waves and *light* waves. In *quantum mechanics*, the wave could be the *wave function* that describes an *electron* propagating according to *Schrödinger's equation* after passing through a narrow slit.

diffraction pattern (B1: 18) The *intensity* pattern of a *wave* that has been diffracted after passing through an aperture or around an obstacle. The *diffraction* arises as a result of *interference*, so the diffraction pattern generally displays *interference maxima* and *interference minima*.

Ehrenfest's theorem (B1: 158) The theorem stating that the *expectation values* of the position and *momentum* of a single *particle* obey the equations

$$\frac{\mathrm{d}\langle x\rangle}{\mathrm{d}t} = \frac{\langle p_x\rangle}{m}$$

and

$$\frac{\mathrm{d}\langle p_x\rangle}{\mathrm{d}t} = -\left\langle \frac{\partial V}{\partial x}\right\rangle,$$

where $V(x)$ is the *potential energy function* and the expectation values on both sides are evaluated in the same *quantum state*. Ehrenfest's theorem is always true. In the *classical limit*, it helps us to understand how the results of *quantum mechanics* approach those of *classical mechanics*.

eigenfunction (B1: 43, 221) A function that satisfies an *eigenvalue equation* together with appropriate *boundary conditions*. In *quantum mechanics*, *eigenfunctions* with different *eigenvalues* are mutually *orthogonal*.

eigenvalue (B1: 43, 221) In the context of *operators* acting on functions, a particular *scalar* λ is an eigenvalue of the operator $\widehat{\mathrm{A}}$ if it permits a solution $f(x)$ to be found to the *eigenvalue equation*

$$\widehat{\mathrm{A}}f(x) = \lambda f(x),$$

where $f(x)$ is not identically equal to zero and is subject to appropriate *boundary conditions*. As a general rule, the eigenvalues of a quantum-mechanical operator $\widehat{\mathrm{A}}$ are the only possible outcomes of a *measurement* of the corresponding *observable*, A.

eigenvalue equation (B1: 43, 221) In the context of *operators* acting on functions of a single variable, an eigenvalue equation is an equation of the form

$$\widehat{\mathrm{A}}f(x) = \lambda f(x),$$

where $\widehat{\mathrm{A}}$ is an operator acting on the function $f(x)$ and the *scalar* λ is an undetermined parameter.

Solving the eigenvalue equation involves finding suitable values of λ and the corresponding functions $f(x)$ that satisfy this equation subject to appropriate *boundary conditions* (e.g. $f(x)$ remaining finite as x tends to $\pm\infty$). The suitable values of λ are the *eigenvalues*, and the corresponding functions are the *eigenfunctions*, of the operator $\widehat{\mathrm{A}}$. The prefix 'eigen' comes from the German for characteristic and indicates the 'special' nature of the eigenfunctions and eigenvalues of an operator.

elastic scattering (B1: 196) *Scattering* in which *kinetic energy* is conserved and *particles* do not change their nature or their *quantum state* of internal excitation; nor are they created, destroyed or absorbed. Contrast with *inelastic scattering*.

electric charge A fundamental property of matter that determines the *electric* and *magnetic forces* between *particles*. There are two types of charge: positive and negative. *Protons* are positively charged (with charge e) and *electrons* are negatively charged (with charge $-e$). The *SI* unit of charge is the coulomb (C) and $e = 1.60 \times 10^{-19}$ C.

electric current The electric current along a wire is the rate of flow of *electric charge* through a fixed plane perpendicular to the axis of the wire. If the current is carried by *electrons* (which are negatively-charged), it is in the opposite direction to the direction of flow of electrons.

electric field A *vector* field that determines the *electric force* on a charged *particle* placed at any given point. The electric field at point P is the electric force per unit *charge* experienced by a small test charge placed at P. The *SI* unit of electric field is $\mathrm{N\,C^{-1}}$.

electric force The force experienced by a charged *particle* due to an *electric field*, given by

$$\mathbf{F} = q\boldsymbol{\mathcal{E}},$$

where q is the *charge* of the particle and $\boldsymbol{\mathcal{E}}$ is the electric field at the position of the particle.

electromagnetic radiation Self-sustaining electromagnetic disturbances that propagate through space with *electric* and *magnetic fields* decreasing no faster than $1/$distance, and the corresponding

energy density decreasing no faster than $1/\text{distance}^2$. Realized through electromagnetic waves.

electron A negatively-charged *particle*, currently regarded as structureless, with about one two-thousandth the mass of a *proton*. Electrons are the carriers of *electric current* in metallic conductors.

electronvolt (B1: 11) A unit of *energy* defined as the *magnitude* of the energy gained when an *electron* is accelerated through a potential difference of one volt. An electronvolt is given the symbol eV, and is equal to 1.60×10^{-19} J.

According to the conventional rules for *SI* prefixes, $1\,\text{meV} = 10^{-3}$ eV, $1\,\text{keV} = 10^{3}$ eV, $1\,\text{MeV} = 10^{6}$ eV and $1\,\text{GeV} = 10^{9}$ eV. The units meV, eV, MeV and GeV are convenient for discussing molecular rotations, electronic transitions, nuclear physics and *elementary particle* physics, respectively.

electrostatic force The force between stationary charged *particles*. For two particles in a vacuum, this force is given by *Coulomb's law*.

electrostatic potential energy *Potential energy* due to *electrostatic forces*. For two *particles* in a vacuum, the electrostatic potential energy is given by the *Coulomb potential energy function*.

element (i) An abbreviation for chemical element. Traditionally, an element is a substance that cannot be divided by chemical means, heating or the passage of an *electric current*. More precisely, a sample of any given element consists of matter entirely composed of *atoms* with the same *atomic number*, and hence the same number of *protons* in their *nuclei*.

(ii) In mathematics, an element is a basic part of something (e.g. a volume element).

elementary particle A piece of matter that is smaller than a *nucleus*. Such *particles* include *protons* and *neutrons*, as well as *electrons* and quarks. They may or may not be truly fundamental constituents of matter.

energy In *classical physics*, energy is the property of a *system* that measures its capacity for doing work on a body, or for changing the *kinetic energy* of a *free particle*. Energy is a *scalar* quantity that is conserved in any *isolated system*. The *SI* unit of energy is the joule, represented by the symbol J, where $1\,\text{J} = 1\,\text{N}\,\text{m} = 1\,\text{kg}\,\text{m}^2\,\text{s}^{-2}$.

energy eigenfunction (B1: 56) An *eigenfunction* of the *Hamiltonian operator* for a given *system*, and therefore a solution of the *time-independent Schrödinger equation* for the system. If the *quantum state* of a system is described by a particular energy eigenfunction, any *measurement* of the *energy* of the system is certain to give the corresponding *energy eigenvalue*.

energy eigenvalue (B1: 56) An *eigenvalue* of the *Hamiltonian operator* for a given *system*. Each energy eigenvalue is an allowed *energy* of of the system.

energy level (B1: 10) An *energy* that characterizes a particular *quantum state* of a *system*. In *bound systems*, such as *nuclei, atoms* and *molecules*, the energy levels are discrete. When such systems become unbound, as in the case of an *ionized* atom, the energy levels form a *continuum*. Do not confuse energy levels with quantum states: if an energy level is *degenerate* it corresponds to more than one quantum state.

energy quantization (B1: 70) The quantum phenomenon in which *bound states* have discrete *energy levels*.

equation of continuity (B1: 189) An equation used in the description of fluid flow that relates changes in the density of fluid in any small region to currents carrying fluid through the boundaries of that region. In *wave mechanics*, the *probability density* and *probability current* obey a similar equation of continuity.

Euler's formula (B1: 214) The relationship

$$\mathrm{e}^{\mathrm{i}\theta} = \cos\theta + \mathrm{i}\sin\theta$$

that links the exponential function to the cosine and sine functions.

even function (B1: 78) A function $f(x)$ for which $f(-x) = f(x)$ for all x. Contrast with *odd function*.

excited state A *quantum state* of a given *system* with an *energy* that is greater than the *ground-state* energy of the system.

expectation value (B1: 111, 229) For a given *random* variable A, the expectation value $\langle A\rangle$ is the theoretical prediction for the *average value* of A in the limiting case of a large number of *measurements*.

Given a discrete set of possible outcomes, $A_1, A_2, \ldots,$ with *probabilities* $p_1, p_2, \ldots,$ the expectation value of A is given by

$$\langle A\rangle = \sum_i p_i A_i,$$

where the sum runs over all the possible values for A.

For a random variable x with a continuous set of possible outcomes, the expectation value is given by

$$\langle x\rangle = \int_{-\infty}^{\infty} \rho(x)\, x\, \mathrm{d}x,$$

where $\rho(x)$ is the *probability density function* for x.

In *quantum mechanics*, the expectation value of an *observable* A is the quantum-mechanical prediction for the average value of A when measurements are taken on a large number of identical *systems* all prepared in the same *quantum state*. In one-dimensional *wave mechanics*, the expectation value of A is given by the *sandwich integral rule*

$$\langle A\rangle = \int_{-\infty}^{\infty} \Psi^*(x,t)\widehat{\mathrm{A}}\Psi(x,t)\,\mathrm{d}x,$$

where $\widehat{\mathrm{A}}$ is the quantum-mechanical *operator* representing the observable A and $\Psi(x,t)$ is the *wave function* of the system at the instant of measurement.

exponential form (B1: 214) A *complex number* is said to be expressed in exponential form if it is written as

$$z = r\mathrm{e}^{\mathrm{i}\theta},$$

where the *real numbers* $r \geq 0$ and θ are called respectively the *modulus* and *phase* of the complex number. Any complex number can be expressed in this form. See also *Cartesian form* and *polar form*.

exponential law of radioactive decay (B1: 13) The physical law stating that, given a large sample of *nuclei* of a given type, the average number of nuclei that remain undecayed at time t falls exponentially as

$$N(t) = N(0)\,\mathrm{e}^{-\lambda t},$$

where $N(0)$ is the initial number of nuclei at time $t = 0$ and λ is the *decay constant* for the given type of nucleus.

F-centre (B1: 83) A defect in an *ionic crystal* (such as NaCl) in which an ejected negative *ion* is replaced by an *electron* that acts as a *particle* trapped in a three-dimensional box created by the surrounding ions. F-centres can absorb one or more *wavelengths* of *visible light* when the trapped electron makes an upward transition between discrete *energy levels*; if present in sufficient numbers, F-centres can alter the colour of a crystal.

finite barrier Any member of a class of *potential energy functions* that are finite everywhere, possess a local maximum, and approach a lower constant value (usually taken to be zero) at large distances from the maximum.

finite square barrier (B1: 181) In one dimension, a *potential energy function* characterized by a constant value of *potential energy* (usually positive) over a finite continuous region of space, with a smaller constant value (usually zero) elsewhere.

The concept can be generalized to two and three dimensions. In this case, the squareness of barrier describes the abruptness of the change in potential energy; a square barrier may occupy any region in space (e.g. a cube or a sphere in three dimensions).

finite square step (B1: 184) In one dimension, a *potential energy function* characterized by a constant value of *potential energy* over some semi-infinite region of space, with a different constant potential energy elsewhere.

finite square well (B1: 85) In one dimension, a *potential energy function* characterized by a constant value of *potential energy* (usually negative) over a finite continuous region of space, with a larger constant value (usually zero) elsewhere. Such a well has a finite number of *bound states* (≥ 1).

The concept can be generalized to two and three dimensions. In this case, the squareness of well describes the abruptness of the change in potential energy; a square well may occupy any region in space (e.g. a cube or a sphere in three dimensions).

finite well (B1: 56) Any member of a class of *potential energy functions* that are finite everywhere, possess a local minimum, and approach a higher constant value (usually taken to be zero) at large distances from the minimum.

first-order partial derivative (B1: 224) A function obtained by taking the *partial derivative* of a given function. Given a function $f(x,y)$ of two variables, we can define two first-order partial derivatives: $\partial f/\partial x$ is the rate of change of f with respect to x when y is held constant, and $\partial f/\partial y$ is the rate of change of f with respect to y when x is held constant.

flux (B1: 196) A quantity that describes the rate of flow of *particles* around a given point in three-dimensional space. The flux is the rate of flow of particles, per unit time per unit area, through a tiny area centred on the given point and perpendicular to the direction of flow of particles at the point. The *SI* unit of flux is $\mathrm{m^{-2}\,s^{-1}}$. Compare with *beam intensity*.

force constant (B1: 125) The positive constant C that appears in *Hooke's law*, $F_x = -Cx$, and in the expression for the *potential energy function* of a *simple harmonic oscillator*, $V(x) = \frac{1}{2}Cx^2$. The *SI* unit of a force constant is $\mathrm{N\,m^{-1}}$.

Fourier transform (B1: 171) The Fourier transform $A(k)$ of a function $f(x)$ is defined by

$$A(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\,\mathrm{e}^{-\mathrm{i}kx}\,\mathrm{d}x.$$

In one dimension, the *momentum amplitude* $A(k)$ of a *free particle* is the Fourier transform of the initial *wave function*, $\Psi(x,0)$. More generally, the momentum amplitude $A(k,t)$ of any *particle* at time t can be taken to be the Fourier transform of the wave function, $\Psi(x,t)$. See also *inverse Fourier transform*.

frame of reference A set of coordinate axes and synchronized clocks, which makes it possible to specify uniquely the location in space and time of any given event.

free particle (B1: 166) A *particle* that is subject to no forces, and for which the *potential energy* is a constant independent of position (usually set equal to zero).

frequency (B1: 10, 17) The number of *cycles* per unit time of an *oscillation* or *wave*. The frequency f is related to the *period* T by $f = 1/T$. The *SI* unit of frequency is the *hertz* ($1\,\mathrm{Hz} = 1\,\mathrm{s^{-1}}$). Compare with *angular frequency*.

fusion (B1: 205) An abbreviation for *nuclear fusion*.

Gaussian function (B1: 131) A function of the form

$$f(x) = C_0\,\mathrm{e}^{-x^2/2a^2},$$

where C_0 and a are constants. The *energy eigenfunctions* of a *harmonic oscillator* all involve Gaussian functions.

Gaussian wave packet (B1: 182) A *wave packet* for which the square of the *modulus* of the *wave function*, $|\Psi|^2$, is shaped like a *Gaussian function* (for a suitable choice of origin).

Geiger counter A device for detecting *particles* that are able to cause the *ionization* of *molecules* in a gas. Also called a Geiger–Müller tube.

Geiger–Nuttall relation (B1: 203) A relation linking the *decay constant* λ of a *nucleus* that emits *alpha particles* to the alpha-particle emission *energy* E_α of that nucleus. The relation applies to specific families of nuclei (such as *isotopes* of a given nucleus) and may be written in the form

$$\lambda = A\mathrm{e}^{-B/E_\alpha^{1/2}},$$

where A is a constant that characterizes the particular family of nuclei, and B depends on the *charge* of the individual nucleus.

general solution (B1: 218) For *differential equations*, the general solution is an expression that describes the general family of functions that satisfy the differential equation. If the *order of a differential equation* is n, the general solution of the equation contains n *arbitrary constants*.

ground state (B1: 10) The *quantum state* of lowest *energy* in a given *system*. In the case of a system that displays *degeneracy*, there may be more than one such state. Contrast with *excited state*.

half-life (B1: 14) For a given type of unstable *nucleus* or unstable *particle*, the time over which half the nuclei or particles, on average, decay. The half-life $T_{1/2}$ is related to the *decay constant* λ by $T_{1/2} = \ln 2/\lambda$.

half-silvered mirror (B1: 15) A mirror coated in such a way that if many *photons* fall on any part of it, at an angle of incidence of 45°, half are transmitted and half are reflected. A single photon incident on a half-silvered mirror emerges in a *linear superposition* of two *states* corresponding to transmission and reflection, and the *probability amplitudes* for these states have equal *magnitudes*. A half-silvered mirror is an example of a *beam splitter*.

Hamiltonian function (B1: 47) A function which, in *classical mechanics*, expresses the total *energy* of a *system* as a sum of (i) the *kinetic energy* expressed in terms of *momentum*, and (ii) the *potential energy function* of the system.

Hamiltonian operator (B1: 48) The quantum-mechanical *operator* corresponding to the total *energy* of a *system*, obtained from the classical *Hamiltonian function* of the system by replacing each *observable* quantity by the corresponding quantum-mechanical operator.

The Hamiltonian operator appears in *Schrödinger's equation*, the *time-independent Schrödinger equation* and in quantum mechanical expressions for the *expectation value* and *uncertainty* of the energy.

harmonic oscillator (B1: 124) A *system* described by a classical *Hamiltonian function* of the form

$$H = \frac{p_x^2}{2m} + \tfrac{1}{2}Cx^2 = \frac{p_x^2}{2m} + \tfrac{1}{2}m\omega_0^2x^2,$$

where m is the mass of the oscillating *particle*, x is its displacement, p_x is its *momentum*, C is the *force constant* and ω_0 is the *angular frequency*.

In *classical physics*, the *restoring force* on the oscillating particle is proportional to its displacement, and the resulting *simple harmonic motion* is *sinusoidal* in time.

In *wave mechanics*, a harmonic oscillator obeys the *Schrödinger equation*

$$\mathrm{i}\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + \tfrac{1}{2}m\omega_0^2x^2\,\Psi(x,t)$$

where ω_0 is now called the classical angular frequency.

Heisenberg uncertainty principle (B1: 117) The principle asserting that, in all *systems* and all *states*, the *uncertainties* of the position and *momentum* components of a *particle* obey the inequality

$$\Delta x\,\Delta p_x \geq \frac{\hbar}{2},$$

where $\hbar$ is *Planck's constant* divided by 2π.

helium atom An *atom* with *atomic number* $Z = 2$, and therefore with two *protons* in its *nucleus*, and two *electrons* outside the nucleus. A helium-4 atom has two *neutrons* in its nucleus (an *alpha particle*) while the rarer *isotope* helium-3 has only one neutron in its nucleus.

Hermite polynomial (B1: 134) A polynomial $H_n(x/a)$ in x/a that appears in the *energy eigenfunctions* of a *harmonic oscillator*:

$$\psi_n(x) = C_nH_n(x/a)\,\mathrm{e}^{-x^2/2a^2},$$

where a is the *length parameter* of the oscillator and C_n is a *normalization constant*.

The nth-order Hermite polynomial $H_n(x)$ in x is defined by

$$H_n(x) = \mathrm{e}^{x^2/2}\left(x - \frac{\mathrm{d}}{\mathrm{d}x}\right)^n \mathrm{e}^{-x^2/2} \quad \text{for } n = 0, 1, 2, \ldots.$$

hertz The *SI* unit of *frequency*, represented by the symbol Hz. A frequency of 1 Hz is equivalent to one *cycle* per second, so $1\,\mathrm{Hz} = 1\,\mathrm{s}^{-1}$.

Hooke's law (B1: 125) The force on a *particle* is said to obey Hooke's law if it is a *restoring force* (always acting towards the equilibrium position) and is proportional to the displacement of the particle from equilibrium. In one dimension, we write

$$F_x = -Cx,$$

where x is the displacement from the equilibrium position, and the proportionality constant C is called the *force constant*. In *classical physics*, such a force leads to *simple harmonic motion*.

hydrogen atom An *atom* with *atomic number* $Z = 1$ and therefore with one *proton* in its *nucleus* and a single *electron* outside the nucleus. Different *isotopes* of hydrogen have no *neutrons*, one neutron (*deuterium*) or two neutrons (tritium). Hydrogen *molecules* consist of two hydrogen atoms bound together.

hyperbolic functions (B1: 220) Functions that involve certain combinations of e^x and e^{-x}. For each trigonometric function involving a given combination of $\mathrm{e}^{\mathrm{i}x}$ and $\mathrm{e}^{-\mathrm{i}x}$, there corresponds a hyperbolic function with the same combination of e^x and e^{-x}. The hyperbolic cosine and hyperbolic sine functions are:

$$\cosh x = \frac{\mathrm{e}^x + \mathrm{e}^{-x}}{2}$$

$$\sinh x = \frac{\mathrm{e}^x - \mathrm{e}^{-x}}{2}.$$

identity operator (B1: 40) An *operator* $\widehat{\mathrm{I}}$ that does not change the object on which it acts. Thus $\widehat{\mathrm{I}}f(x) = f(x)$.

imaginary axis (B1: 212) An axis in the *complex plane* along which *complex numbers* have zero *real part* and which points in the direction of increasing *imaginary part*.

imaginary number (B1: 210) A *complex number* of the form $\mathrm{i}y$, where y is *real*. Compare with *imaginary part*.

imaginary part (B1: 210) Given a *complex number* $z = x + \mathrm{i}y$, where x and y are *real numbers*, the imaginary part of z is equal to y. The imaginary part of any complex expression z is given by

$$\mathrm{Im}(z) = \frac{z - z^*}{2\mathrm{i}},$$

where z^* is the *complex conjugate* of z. Note that the imaginary part is a real number and does not include a factor of i.

in phase Two *oscillations* or *waves* are said to be in phase with one another if they permanently have the same *phase*.

independent variable In a function $f(x, y, \ldots, z)$, the independent variables are $x, y, \ldots, z$. Contrast with the *dependent variable* f.

indeterministic (B1: 13) A term indicating that identical *initial conditions* determine only the *probabilities* of various possible outcomes rather than guaranteeing the same outcome on all occasions. Indeterminism is a key feature of *quantum mechanics*. Contrast with *deterministic*.

inelastic scattering (B1: 196) *Scattering* for which *kinetic energy* is not conserved. In inelastic scattering, *particles* may change their *quantum state* of internal excitation or be absorbed; they may even be created or destroyed, especially at very high *energies*. Contrast with *elastic scattering*.

inertial frame of reference (B1: 127) A *frame of reference* set up and moving in such a way that all *free particles* have zero acceleration, in accordance with Newton's first law. The laws of physics (including those of *classical mechanics* and *quantum mechanics*) are normally expressed in terms of observations made in inertial frames of reference.

infinite square well A *potential energy function* that is infinite everywhere except for a finite region, within which the potential energy has a constant value (which is usually taken to be zero).

In two and three dimensions, the region where the potential energy function is finite and constant may be of any shape; the 'squareness' refers to the abrupt discontinuity in the potential energy function not to the shape of the region in space. See *one-dimensional infinite square well*, *two-dimensional infinite square well* and *three-dimensional infinite square well*.

infinite well A *potential energy function* that has a minimum and increases to infinity in all directions away from the minimum. Examples include *infinite square wells* and the potential energy function of a *harmonic oscillator*. All the *energy eigenvalues* in an infinite well are discrete and all the *energy eigenfunctions* describe *bound states*.

initial conditions (B1: 221) Conditions that supply extra information about the solution of a *differential equation* at a single value of the *independent variable*.

intensity of a beam Another term for *beam intensity*. Compare with the *intensity of a wave*.

intensity of a wave For a classical *wave*, intensity is the amount of *energy* transported by the wave per unit time per unit area perpendicular to the direction of wave propagation. Intensity is proportional to the square of the *amplitude* of the wave. If a point source of waves radiates energy equally in all directions, the intensity of the waves is inversely proportional to the square of the distance from the source. The *SI* unit of intensity is *watt* per square metre ($\mathrm{W\,m^{-2}}$).

interference (B1: 16) A phenomenon arising from the superposition of two or more *coherent waves*, resulting in a pattern of *interference maxima*

and *interference minima*. See also *constructive interference* and *destructive interference*.

interference maxima (B1: 18) Features of *interference patterns* produced when two or more *coherent waves* interfere with one another. Interference maxima occur at points where the contributing waves reinforce one another, leading to a local maximum in the *intensity* of the wave (*constructive interference*).

interference minima (B1: 18) Features of *interference patterns* produced when two or more *coherent waves* interfere with one another. Interference minima occur at points where the contributing waves cancel one another, leading to a local minimum in the *intensity* of the wave (*destructive interference*).

interference pattern A pattern of peaks and troughs in *intensity* that occurs when *coherent waves* from a spatially extended source (or more than one source) are allowed to interfere with one another. See *interference*.

interference rule (B1: 32) A rule stating that, if a given process, leading from an initial *quantum state* to a final quantum state, can proceed in two or more alternative ways, and the way taken is not recorded, the *probability amplitude* for the process is the sum of the probability amplitudes for the different ways. When calculating the *probability* of the process, it is essential to add all the contributing probability amplitudes before taking the square of the *modulus*.

inverse Fourier transform (B1: 171) An operation that is the inverse of the *Fourier transform*. The inverse Fourier transform $f(x)$ of a function $A(k)$ is defined as

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(k)\, \mathrm{e}^{\mathrm{i}kx}\, \mathrm{d}k.$$

The initial *wave function* $\Psi(x, 0)$ of a *free particle* is the inverse Fourier transform of the *momentum amplitude* $A(k)$.

ion (B1: 12) An electrically-charged *atom* formed when a neutral atom loses or gains one or more *electrons* so that the *magnitude* of its total electronic *charge* is not equal to the magnitude of the charge of the *nucleus*.

ionic crystal (B1: 83) A crystal composed mainly of positive and negative *ions* held together by attractive *electrostatic forces* between ions of opposite sign. A familiar example is the crystal of common salt, NaCl.

ionization (B1: 11) The process in which a neutral *atom* loses one or more *electrons* to produce a positive *ion* and one or more unbound electrons.

ionized (B1: 11) The condition of an *atom* that has lost one or more *electrons*.

isolated system A *system* that is free from external influences. Such a system is not acted upon by external forces and exchanges no *energy* or matter with the rest of the Universe.

isotope (B1: 13) Isotopes of a given *element* are *atoms* or *nuclei* with the same number of *protons*, and therefore the same *atomic number*, Z. Different isotopes have different numbers of *neutrons*, and therefore different *mass numbers*, A.

The usual symbol for an isotope is ${}^{A}\mathrm{Sy}$, where Sy is the chemical symbol for the element. Since Sy determines Z, the fuller notation ${}^{A}_{Z}\mathrm{Sy}$ is strictly redundant, but may be helpful for elements for which Z is not widely-remembered. For example, the isotope of silicon with $A = 27$ and $Z = 14$ can be written as ${}^{27}\mathrm{Si}$ or ${}^{27}_{14}\mathrm{Si}$. Less formally, this isotope is referred to as silicon-27.

kelvin The unit of temperature on the *absolute temperature scale*, represented by the symbol K. This is the *SI* unit of temperature. A temperature difference of one kelvin $(1\,\mathrm{K})$ is equivalent to a temperature difference of one degree Celsius $(1\,^{\circ}\mathrm{C})$, but the two scales have different zeros, with zero kelvin corresponding to *absolute zero*, the lowest conceivable temperature for a *system* in equilibrium.

kinetic energy *Energy* due to motion. The kinetic energy of a *particle* is given by

$$E_{\text{kin}} = \frac{1}{2}mv^2 = \frac{p^2}{2m},$$

where m is the mass of the particle, v is its speed and p is the *magnitude* of its *momentum*.

kinetic energy operator (B1: 44) The quantum-mechanical *operator* representing the *kinetic energy* of a *system*. In one dimension, a *particle* of mass $m \neq 0$ has the kinetic energy operator

$$\widehat{\mathrm{E}}_{\text{kin}} = \frac{\widehat{\mathrm{p}}_x^2}{2m} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}.$$

Kronecker delta symbol (B1: 103) The symbol δ_{ij} that represents 1 if $i = j$, and represents 0 if $i \neq j$. Thus,

$$\sum_i a_i \delta_{ij} = a_j.$$

ladder operator (B1: 140) A term used to describe the *raising operator* or the *lowering operator* of a *harmonic oscillator*.

length parameter (B1: 133) For a *harmonic oscillator*, the length parameter is

$$a = \sqrt{\frac{\hbar}{m\omega_0}},$$

where $\hbar$ is *Planck's constant* divided by 2π, m is the mass of the oscillating *particle*, $\omega_0 = \sqrt{C/m}$ is the classical *angular frequency* and C is the *force constant* of the oscillator.

The length parameter characterizes the quantum properties of the oscillator. For example, in a *state* with *quantum number* n, the *uncertainties* in position and *momentum* are given by $(n+\frac{1}{2})^{1/2}a$ and $(n+\frac{1}{2})^{1/2}\hbar/a$, respectively.

light A term normally reserved for *visible light*, but sometimes used for any type of *electromagnetic radiation*.

linear combination Given a set of functions $f_1(x)$, $f_2(x)$, …, and a set of (possibly *complex*) constants c_1, c_2, …, any expression of the form

$$c_1 f_1(x) + c_2 f_2(x) + \ldots$$

is called a linear combination of $f_1(x)$, $f_2(x)$, ….

Also called *linear superposition*.

linear differential equation (B1: 217) An *ordinary differential equation* of the form

$$\widehat{\mathrm{L}}y(x) = f(x),$$

where $\widehat{\mathrm{L}}$ is a *linear differential operator*.

linear differential operator (B1: 218) An *linear operator* of the form

$$\widehat{\mathrm{L}} = a_n(x)\frac{\mathrm{d}^n}{\mathrm{d}x^n} + a_{n-1}(x)\frac{\mathrm{d}^{n-1}}{\mathrm{d}x^{n-1}} + \ldots + a_1(x)\frac{\mathrm{d}}{\mathrm{d}x} + a_0(x),$$

where $a_n(x), a_{n-1}(x) \ldots a_0(x)$ are functions of x (some of which may be constant or equal to zero). A *linear differential equation* can be written in the form $\widehat{\mathrm{L}}y(x) = f(x)$, where the special case $\widehat{\mathrm{L}}y(x) = \lambda y(x)$ is an *eigenvalue equation*.

linear homogeneous differential equation (B1: 218) An ordinary or partial *linear differential equation* in which each term is proportional to the *dependent variable* or one of its derivatives with respect to the *independent variable*, and there is no term that is a constant or that depends only on the independent variable. For example,

$$\frac{\mathrm{d}^2y(x)}{\mathrm{d}x^2} + \sin(2x)\,y(x) = 0$$

is a linear homogeneous differential equation with dependent variable y and independent variable x.

Eigenvalue equations of the form $\widehat{\mathrm{L}}f(x) = \lambda f(x)$, where $\widehat{\mathrm{L}}$ is a *linear differential operator* are examples of linear homogeneous differential equations. The solutions of linear homogeneous differential equations obey the *principle of superposition*.

linear number density (B1: 183) The number of specified entities (e.g. *atoms*) per unit length, along a given line. The *SI* unit of linear number density is m^{-1}. Do not confuse linear number density with *number density*, which is a different quantity with different units.

linear operator (B1: 42) An *operator* $\widehat{\mathrm{O}}$ that satisfies the condition

$$\widehat{\mathrm{O}}\left[\alpha f(x) + \beta g(x)\right] = \alpha\widehat{\mathrm{O}}f(x) + \beta\widehat{\mathrm{O}}g(x)$$

for arbitrary *complex* constants α and β.

In *quantum mechanics*, all *observable* quantities are represented by linear operators.

linear superposition An alternative term for *linear combination*.

lowering operator (B1: 137) For a *harmonic oscillator* with *length parameter* a, the lowering operator is defined as

$$\widehat{\mathrm{A}} = \frac{1}{\sqrt{2}}\Big(\frac{x}{a} + a\frac{\partial}{\partial x}\Big),$$

For $n \geq 1$, this *operator* converts an *energy eigenfunction* $\psi_n(x)$ of the oscillator into the next eigenfunction of lower *energy*, while $\widehat{\mathrm{A}}\psi_0(x) = 0$. If the eigenfunctions are *normalized*,

$$\widehat{\mathrm{A}}\,\psi_n(x) = \sqrt{n}\,\psi_{n-1}(x) \quad \text{for } n = 1,\, 2,\, \ldots.$$

See also *raising operator* and *ladder operator*.

Mach–Zehnder interferometer (B1: 32) An arrangement of two *half-silvered mirrors*, which act as *beam splitters*, two fully-reflecting mirrors, and two detectors, which is used to demonstrate *interference* phenomena in *light*. These interference phenomena are evident even if only one *photon* passes through the Mach–Zehnder interferometer at a time.

magnetic field A *vector* field which determines the *magnetic force* on a charged *particle* moving through a given point. The magnetic field at a point P is a vector quantity $\mathbf{B}$ such that the magnetic force on a particle of *charge* q, passing through P with velocity $\mathbf{v}$, is given by the vector product

$$\mathbf{F} = q\big(\mathbf{v} \times \mathbf{B}\big).$$

magnetic force A velocity-dependent force experienced by a *particle* with an *electric charge* moving in a *magnetic field*. The force is perpendicular to both the velocity of the particle and the magnetic field, and is given by the vector product

$$\mathbf{F} = q\big(\mathbf{v} \times \mathbf{B}\big),$$

where q is the particle's *charge*, $\mathbf{v}$ is its velocity and $\mathbf{B}$ is the magnetic field at the position of the particle.

magnitude A non-negative *real* quantity. The magnitude of a *scalar* Q is a non-negative quantity $|Q|$ describing the size of Q, irrespective of its sign. The magnitude of a *vector* $\mathbf{a}$ is a non-negative quantity $|\mathbf{a}|$, describing the size of $\mathbf{a}$, irrespective of its direction; this is usually written as a, omitting both the bold print and the modulus signs.

mass number The total number of *neutrons* and *protons* in a *nucleus*, $A = N + Z$, where N is the number of neutrons and Z is the number of protons (the *atomic number*).

Maxwell's equations (B1: 17) A set of four *partial differential equations* relating the *electric field* $\mathbf{E}(\mathbf{r}, t)$ and *magnetic field* $\mathbf{B}(\mathbf{r}, t)$ to each other and to their sources in *electric charges* and currents. These are the basic field equations of classical electromagnetism.

mean value (B1: 111, 228) An alternative term for *average value*.

measurement In *quantum mechanics*, a measurement is an interaction or communication of information between a *quantum system* and a classical measuring device. The measurement occurs when a quantum system causes some sort of irreversible change in the measuring device, and possibly in its surroundings, and the system undergoes a *collapse of the wave function*.

modulus (B1: 210) For any *complex number* $z = x + \mathrm{i}y$, the modulus is the non-negative *real number*

$$|z| = \sqrt{x^2 + y^2},$$

where x is the *real part* and y is the *imaginary part* of z. If the complex number is written in *polar form* $z = r(\cos\theta + \mathrm{i}\sin\theta)$ or in *exponential form* $z = r\mathrm{e}^{\mathrm{i}\theta}$, the modulus is equal to r. For any complex expression, $|z| = \sqrt{zz^*}$.

molecule Traditionally, the smallest part of a pure substance that retains the chemical identity of that substance. From a microscopic point of view, a molecule is a particular group of *atoms* bound together in a particular way.

momentum A *vector* quantity describing of the amount of translational motion of a *particle* or *system*. In *classical mechanics*, the momentum of a particle is

$$\mathbf{p} = m\mathbf{v},$$

where m is the particle's mass and $\mathbf{v}$ is its velocity. The momentum of a system of particles is the vector sum of the momenta of all the particles. The *SI* unit of momentum is $\mathrm{kg\,m\,s^{-1}}$.

momentum amplitude (B1: 170) A quantity whose *modulus* squared gives the *probability distribution* of *momentum* via *Born's rule for momentum*.

For a *free particle* in one dimension, the momentum amplitude $A(k)$ is the function that appears in the expansion of a free-particle *wave function* $\Psi(x, t)$ in terms of *de Broglie wave functions*:

$$\Psi(x,t) = \int_{-\infty}^{\infty} A(k)\,\mathrm{e}^{\mathrm{i}(kx - E_k t/\hbar)}\,\mathrm{d}x.$$

It is given by the *Fourier transform* of the initial free-particle wave function:

$$A(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \Psi(x,0)\mathrm{e}^{-\mathrm{i}kx}\,\mathrm{d}x$$

For non-free particles, the momentum amplitude $A(k, t)$ depends on time and is given by the Fourier transform of the wave function, $\Psi(x, t)$. Also called the *momentum wave function*.

momentum eigenfunction (B1: 168) For a single *particle* in one dimension, an *eigenfunction* of the *momentum operator* conventionally represented as

$$\psi_k(x) = \frac{1}{\sqrt{2\pi}}\mathrm{e}^{\mathrm{i}kx},$$

where k is a *real* constant called the *wave number* and the corresponding *eigenvalue* is $\hbar k$. The factor $1/\sqrt{2\pi}$ is included by convention so that the *Fourier transform* of any function $\phi(x)$ is simply expressed as $\langle\psi_k|\phi\rangle$.

momentum operator (B1: 44) The quantum-mechanical *operator* representing the *momentum* of a *system*. For a single *particle* in one dimension, the momentum operator is

$$\widehat{\mathrm{p}}_x = -\mathrm{i}\hbar\frac{\partial}{\partial x}.$$

momentum wave function (B1: 170) Another term for the *momentum amplitude*.

mutually exclusive outcomes (B1: 228) A set of outcomes such that the occurrence of one of them in a given measurement implies that none of the others occur in that measurement. The *probability* that one or other of a set of mutually exclusive outcomes will occur is found by adding their individual probabilities. This is the *addition rule for probability*.

neutron An electrically-neutral *elementary particle* which is a constituent of all atomic *nuclei* (except the common form of *hydrogen*). A neutron has a mass slightly greater than that of a *proton*, it has no net *charge*, but it does have a magnetic dipole moment. The neutron is stable within an atomic nucleus, but is unstable in a vacuum where it has a *half-life* of $914\,\mathrm{s}$.

nodes (B1: 73) Fixed points of zero disturbance in a *standing wave* (excluding points on the boundaries of the region where the disturbance takes place).

non-degenerate A set of *stationary states* or *energy eigenfunctions* is said to be non-degenerate if all members of the set correspond to different *energies*. An *energy level* that corresponds to only one *quantum state* is also said to be non-degenerate. Contrast with *degenerate*.

normal ordering (B1: 138) The conventional ordering $\widehat{\mathrm{A}}^\dagger\widehat{\mathrm{A}}$ for the *raising* and *lowering operators* of a *harmonic oscillator*. These *operators* do not *commute* with one another so their order matters. In fact, we have $\widehat{\mathrm{A}}\widehat{\mathrm{A}}^\dagger = \widehat{\mathrm{A}}^\dagger\widehat{\mathrm{A}} + \widehat{\mathrm{I}}$, where $\widehat{\mathrm{I}}$ is the *identity operator*.

normalization condition (B1: 50) A condition, based on *Born's rule*, which requires that the *probability* of finding a given *particle* to be somewhere in the whole of space is equal to 1. For a

single particle in one dimension, in a *state* described by the *wave function* $\Psi(x,t)$, the normalization condition takes the form

$$\int_{-\infty}^{\infty} |\Psi(x,t)|^2 \, dx = 1.$$

By convention, *bound state energy eigenfunctions* $\psi_n(x)$ also obey a normalization condition:

$$\int_{-\infty}^{\infty} |\psi_n(x)|^2 \, dx = 1.$$

Similar normalization conditions apply to *systems* of more than one particle in more than one dimension.

normalization constant (B1: 50) A constant A chosen so that, if $\Phi(x,t)$ is a *normalizable wave function*, then $\Psi(x,t) = A\Phi(x,t)$ is a *normalized* wave function. Because any overall *phase factor* multiplying a wave function has no physical significance, the normalization constant of a one-dimensional *system* may be taken to be the *real* positive constant

$$A = \left(\int_{-\infty}^{\infty} |\Phi(x,t)|^2 \, dx \right)^{-1/2}.$$

This turns out to be independent of time. This result can be extended to *energy eigenfunctions* and to systems of more than one *particle* in more than one dimension.

normalization integral In one dimension, an integral of the form

$$\int_{-\infty}^{\infty} |f(x)|^2 \, dx$$

that must be set equal to 1 to ensure that the function $f(x)$ is *normalized*.

normalization rule (B1: 228) A mathematical rule setting the scale for the *probabilities* of outcomes in a discrete *complete set* of *mutually exclusive outcomes*. The probabilities obey the rule

$$\sum_i p_i = 1,$$

where the sum is over all the outcomes in the set.

This condition can be extended to a *real* variable x with a *continuous probability distribution*. In this case,

$$\int_{-\infty}^{\infty} \rho(x) \, dx = 1,$$

where $\rho(x)$ is the *probability density function* for the variable x.

normalized (B1: 50) A term used to describe a *wave function* or *eigenfunction* that satisfies the *normalization condition*.

nuclear fusion A process in which two *nuclei* fuse together to form a larger nucleus. If the final nucleus has *mass number* $A \leq 56$ or thereabouts (e.g. ^{56}Fe), the mass of the final nucleus is less than the sum of the masses of the initial nuclei, and *energy* is released. Fusion reactions often proceed by quantum-mechanical *tunnelling*.

In stars, fusion is crucial for energy release and for the conversion of primordial *hydrogen* into other light *elements*, from *helium* up to iron. Fusion reactions are also the basis for the energy released in hydrogen bombs.

nuclei The plural of *nucleus*.

nucleus The positively-charged, very compact central part of an *atom*, composed of *protons* and *neutrons*. The nucleus is some 10^4 times smaller in radius than an atom, but contains nearly all the mass. The number of positively-charged protons in the nucleus of a given *element* is equal to the *atomic number*, Z.

number density The number of specified entities (e.g. *atoms*) per unit volume.

number operator (B1: 141) The *operator* $\widehat{A}^\dagger\widehat{A}$, where $\widehat{A}^\dagger$ is the *raising operator* and $\widehat{A}$ is the *lowering operator* for a *harmonic oscillator*. When the number operator acts on an *energy eigenfunction* of the harmonic oscillator, it gives the *eigenvalue equation*

$$\widehat{A}^\dagger\widehat{A}\, \psi_n(x) = n\, \psi_n(x),$$

where $n = 0, 1, 2, \ldots$ is the *quantum number* of the eigenfunction $\psi_n(x)$.

observable (B1: 44) Any quantity that can be measured. In *quantum mechanics*, each observable quantity is represented by a *linear operator*.

odd function (B1: 78) A function $f(x)$ for which $f(-x) = -f(x)$ for all x. Compare with *even function*.

old quantum theory (B1: 7) A term given to attempts made between 1900 and 1925 to reconcile quantum concepts with *classical physics*. The resulting theory was based on ad-hoc ideas and never became a comprehensive world-view. While it had successes (the Bohr model explained the *spectral lines* of *hydrogen atoms*) it ran into serious difficulties (failing to explain the spectral lines of *helium atoms*, for example). Old quantum theory was superseded by *quantum mechanics* in the period from 1924 to 1927.

one-dimensional infinite square well (B1: 65) A one-dimensional *potential energy function* that is infinite everywhere except for a finite region, within which the *potential energy* has a constant value (usually taken to be zero). In a one-dimensional infinite square well of width L, with zero potential energy within the well, a *particle* of mass m has *energy eigenvalues*

$$E_n = \frac{n^2\hbar^2}{2mL^2},$$

where the *quantum number* n is a positive integer. The *energy eigenfunctions* are *sinusoidal* functions and there is no *degeneracy* in this case.

operator (B1: 40) In the context of functions, an operator is a mathematical entity that acts on a function to produce another function. It is conventional to indicate an operator by putting a hat on it, as in $\widehat{\mathrm{O}}$. Examples of such operators include the *momentum operator* $\widehat{\mathrm{p}}_x = -\mathrm{i}\hbar\partial/\partial x$, the *position operator* $\widehat{\mathrm{x}} = x$ and the *Hamiltonian operator*, $\widehat{\mathrm{H}}$, which represents the total *energy* of the *system*.

order of a derivative The number of times a function is differentiated. For example, $\mathrm{d}^3y/\mathrm{d}x^3$ is a third-order derivative.

order of a differential equation (B1: 217) The highest *order of derivative* that appears in the *differential equation*.

ordinary differential equation (B1: 217) A *differential equation* that involves only ordinary derivatives (and no *partial derivatives*). Sometimes, for brevity, called a differential equation.

orthogonal functions (B1: 102) Two functions $f(x)$ and $g(x)$ are said to be orthogonal if their *overlap integral* vanishes. That is,

$$\int_{-\infty}^{\infty} f^*(x)\, g(x)\, \mathrm{d}x = 0.$$

This definition can be extended to three dimensions by taking the integral to be over all space.

orthonormal set of functions (B1: 103) A set of functions $\psi_1(x), \psi_2(x), \ldots$ is said to be orthonormal if each function is *normalized* and any pair of functions is *orthogonal*:

$$\int_{-\infty}^{\infty} \psi_i^*(x)\, \psi_j(x)\, \mathrm{d}x = \delta_{ij},$$

where δ_{ij} is the *Kronecker delta symbol*. This definition can be extended to three dimensions by taking the integral to be over all space.

oscillation A to-and fro motion, also called a vibration.

overlap integral (B1: 101) In one dimension, the overlap integral of two functions $f(x)$ and $g(x)$ is given by

$$\int_{-\infty}^{\infty} f^*(x)\, g(x)\, \mathrm{d}x.$$

This definition can be extended to three dimensions by taking the integral to be over all space. Overlap integrals are used to calculate *probability amplitudes* in *wave mechanics*.

overlap rule (B1: 101) For a one-dimensional *system* with discrete *non-degenerate energy eigenvalues* $E_1, E_2, \ldots$, the overlap rule states that the *probability* of obtaining the ith *eigenvalue* E_i is

$$p_i = \left| \int_{-\infty}^{\infty} \psi_i^*(x)\, \Psi(x,t)\, \mathrm{d}x \right|^2,$$

where $\psi_i(x)$ is the *normalized energy eigenfunction* with eigenvalue E_i, and $\Psi(x,t)$ is the normalized *wave function* describing the *state* of the system at the time of *measurement*. The integral between the *modulus* signs is called the *overlap integral* of $\psi_i(x)$ and $\Psi(x,t)$.

This rule can be extended to other *observables* with discrete eigenvalues, provided that the energy eigenfunctions are replaced by the eigenfunctions of the corresponding quantum-mechanical *operator*.

Parseval's theorem A term sometimes used in physics and engineering for *Plancherel's theorem*. Historically, Parseval proved a precursor of Plancheral's theorem for sums of *sinusoidal* functions (Fourier series) and it was Plancherel who proved the corresponding theorem for integrals (*Fourier transforms*).

partial derivative (B1: 222) The partial derivative $\partial f/\partial x$ of a function $f(x, y, \ldots)$ with respect to the *independent variable* x is the derivative obtained by differentiating f with respect to x while treating all the other variables as constants.

partial differential equation (B1: 224) An equation that involves *partial derivatives* of an unknown function. See also *differential equation*.

partial differentiation The process of differentiating a function of two or more variables with respect to one of its variables, while treating all the other variables as constants.

particle (i) In the context of *classical physics*, a particle is an idealized object that is thought of as existing at a single point in space. It has no size, shape or internal motion though it may have intrinsic properties such as mass and *charge*, as well as position, velocity and acceleration.

(ii) In the context of high-energy physics, a particle is a piece of matter that is of sub-nuclear size (an *elementary particle*). Such particles include *protons*, *neutrons* and *electrons*, and may or may not be truly fundamental constituents of matter.

particle decay A general process whereby an unstable *elementary particle* can spontaneously change into two or more other elementary particles.

particular solution (B1: 218) A particular solution of a *differential equation* is a specific function that satisfies the equation and does not contain *arbitrary constants*. A particular solution is obtained from the *general solution* of the differential equation by choosing values for all the arbitrary constants. The

choice is usually made by imposing additional physical restraints (*boundary conditions* or *initial conditions*).

period (B1: 17) The time T taken for one complete *cycle* of an *oscillation* or *wave*; the reciprocal of the *frequency*: $T = 1/f$.

permittivity of free space (B1: 204) The fundamental constant ε_0 that appears in the proportionality factor $1/4\pi\varepsilon_0$ for *Coulomb's law* and hence determines the *magnitude* of the *electrostatic force* between two *electric charges* separated by a fixed distance in a vacuum. It also appears in the electrostatic *potential energy function* for two charges. In *SI* units, $\epsilon_0 = 8.85\times10^{-12}\,\mathrm{F\,m^{-1}} = 8.85\times10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$, where F is the symbol for farad.

phase (B1: 18, 213) (i) For a *sinusoidal oscillation* $x(t) = A\cos(\omega t + \phi)$, where A is positive, the phase is the *argument* of the cosine, i.e. $\omega t + \phi$.

For a sinusoidal *wave* $u(x,t) = A\cos(kx - \omega t + \phi)$, where A is positive, the phase is the argument of the cosine, i.e. $kx - \omega t + \phi$. Do not confuse the phase with the *phase constant*.

(ii) For a *complex number* written in *polar* or *exponential form*:

$$z = r(\cos\theta + \mathrm{i}\sin\theta) = r\mathrm{e}^{\mathrm{i}\theta},$$

the phase of the complex number is the *real number* θ (also called the argument of the complex number).

(iii) The phase of a substance is a physically distinctive form of the substance that normally exists over a range of pressures and temperatures, such the solid, liquid and gas phases of water, or the magnetic or superconducting phases of a metal.

phase constant (B1: 18, 126) For a *sinusoidal oscillation* $x(t) = A\cos(\omega t + \phi)$, where A is positive, the phase constant is ϕ.

For a sinusoidal *wave* $u(x,t) = A\cos(kx - \omega t + \phi)$, where A is positive, the phase constant is ϕ. Do not confuse the phase constant with the *phase*.

phase factor (B1: 52, 214) A *complex number* of the form $\mathrm{e}^{\mathrm{i}\alpha}$, where α is a *real* number called the *phase*. The *modulus* of a phase factor is equal to 1. Multiplying a *wave function* by an overall phase factor has no physical significance.

photon (B1: 10) A packet of *electromagnetic radiation*. For radiation in a vacuum, with *frequency* f and *angular frequency* ω, each photon has *energy* $E = hf = \hbar\omega$ and *momentum* of *magnitude* $p = hf/c = \hbar\omega/c$, where h is *Planck's constant*, $\hbar$ is Planck's constant divided by 2π and c is the *speed of light* in a vacuum. Photons are emitted and absorbed in *radiative transitions* between *energy levels*.

Plancherel's theorem (B1: 169) A mathematical theorem stating that

$$\int_{-\infty}^{\infty} |f(x)|^2\,\mathrm{d}x = \int_{-\infty}^{\infty} |A(k)|^2\,\mathrm{d}k,$$

where $A(k)$ is the *Fourier transform* of $f(x)$. This theorem allows us to construct *normalized wave packets* for *free particles*, and underpins the interpretation of $A(k)$ as a *momentum amplitude*. Plancherel's theorem is sometimes called *Parseval's theorem*.

Planck's constant (B1: 10) The fundamental constant $h = 6.63\times10^{-34}\,\mathrm{J\,s}$ that appears in practically every equation of *quantum mechanics*, but never in those of *classical physics*. The quantity $h/2\pi$ is given the symbol $\hbar$.

plane wave A *wave* of constant *frequency* for which points of constant *phase* lie in planes perpendicular to the direction of propagation of the wave.

polar form (B1: 213) The polar form of a *complex number* is

$$z = r(\cos\theta + \mathrm{i}\sin\theta),$$

where r is the *modulus* of the complex number and θ is its *phase* or *argument*. The modulus lies in the range $0 \le r < \infty$. It is always possible to add any integer multiple of 2π to the phase without changing the complex number.

position operator (B1: 45) The quantum-mechanical *operator* representing the position of a *particle*. In one dimension, the position operator is $\widehat{\mathrm{x}} = x$; that is, the operation of multiplying a function $f(x)$ by the variable x.

potential energy *Energy* associated with the position of a *particle* or energy stored in a *system* by virtue of the positions of its component parts. In one dimension, a particle experiencing a conservative force $F_x(x)$ has the *potential energy function*

$$V(x) = -\int F_x(x)\,\mathrm{d}x + \text{constant}.$$

Equivalently,

$$F_x(x) = -\frac{\mathrm{d}V}{\mathrm{d}x}.$$

Any convenient point can be chosen to be the *zero of potential energy*.

potential energy function (B1: 46) A function describing the *potential energy* of a *particle* or *system*. Examples include the *free-particle* potential energy function which is a constant (usually taken to be zero) everywhere, the one-dimensional *harmonic oscillator* potential energy function $V(x) = \frac{1}{2}Cx^2$, and various *finite well* potential energy functions.

potential energy operator (B1: 45) The quantum-mechanical *operator* representing the *potential energy function* of a *system*. For a single *particle* in one dimension, the *potential energy*

operator is $\widehat{\mathrm{V}} = V(x)$; that is, the operation of multiplying a function $f(x)$ by the potential energy function $V(x)$.

principle of superposition (B1: 52, 219) The property of a *linear homogeneous differential equation* whereby, if $y_1(x)$ and $y_2(x)$ are solutions of the differential equation, then so is the *linear combination* $c_1y_1(x) + c_2y_2(x)$, where c_1 and c_2 are any constants. *Schrödinger's equation* is a linear homogeneous *partial differential equation*, so the principle of superposition applies to it.

A version of the principle of superposition applies to *eigenvalue equations* for *linear operators*. If $f(x)$ and $g(x)$ are *eigenfunctions* of the linear operator $\widehat{\mathrm{A}}$ with the same *eigenvalue* λ then, for any constants α and β, the linear combination $\alpha f(x) + \beta g(x)$ is also an eigenfunction of $\widehat{\mathrm{A}}$ with eigenvalue λ.

probabilistic An alternative term for *indeterministic*.

probability (B1: 227) A number between 0 and 1 used to quantify the likelihood of an uncertain outcome, with larger probabilities corresponding to more likely outcomes; impossibility is represented by 0 and certainty by 1. See also the *addition rule for probability* and the *normalization rule*.

probability amplitude (B1: 31, 102) A quantity that emerges from quantum-mechanical calculations and refers to a given experimental outcome. The *probability* of a given outcome is obtained by taking the square of the *modulus* of the corresponding probability amplitude. Probability amplitudes can be calculated using the *overlap rule* or the *coefficient rule* and they are combined using the *interference rule*. Do not confuse a probability amplitude with the *amplitude* of a *wave* or *oscillation*.

probability current (B1: 190) A quantity that describes the rate of flow of *probability density* in one dimension. For a *particle* of mass m, or a beam of particles each of mass m, the probability current is defined by the relation

$$j_x(x,t) = -\frac{\mathrm{i}\hbar}{2m}\left(\Psi^*\frac{\partial\Psi}{\partial x} - \Psi\frac{\partial\Psi^*}{\partial x}\right),$$

where $\Psi(x,t)$ is the *wave function* describing the particle or beam. The probability current can be positive, negative or zero. The *SI* unit of probability current is s^{-1}. See also *beam intensity*.

probability density (B1: 50) For a *particle* in one dimension, in a *quantum state* described by the *wave function* $\Psi(x,t)$, the probability density is given by $|\Psi(x,t)|^2$. This is the *probability* per unit length of finding the particle in a small interval centred on x.

A similar definition applies in three dimensions, where the probability density $|\Psi(\mathbf{r},t)|^2$ is the probability per unit volume of finding the particle in a small region centred on $\mathbf{r}$.

Strictly speaking, the probability density is correctly called the *probability density function* for position.

probability density function (B1: 231) For any continuous *random* variable, a, the probability density function is a function $\rho(a)$, defined such that the *probability* of obtaining a value of a lying in a small range of width δa, centred on a_0 is $\rho(a_0)\,\delta a$. The probability P of finding a value of a between a_1 and a_2 is

$$P = \int_{a_1}^{a_2} \rho(a)\,\mathrm{d}a.$$

The probability density function satisfies the *normalization condition*

$$\int_{-\infty}^{\infty} \rho(a)\,\mathrm{d}a = 1.$$

In *wave mechanics*, the probability density function for position is usually referred to simply as the *probability density*.

probability distribution (B1: 229) For a discrete *random* variable, a function that assigns a *probability* to each possible value of the variable. For a continuous random variable, a function that assigns a *probability density* to each possible value of the variable.

proton An *elementary particle* which is a constituent of all atomic *nuclei*. The mass of a proton is slightly less that of a *neutron* and is almost 2000 times greater than that of an *electron*. The *charge* of a proton is positive and has the same *magnitude* as that of a negatively-charged electron.

proton–proton chain (B1: 206) A sequence of nuclear reactions in stars like the Sun that have the net effect of converting *hydrogen* into *helium*.

quantization (B1: 10) The phenomenon in which the measured values of some *observable* quantities have a discrete set of allowed values (in given *systems*, over given ranges).

quantum dot (B1: 63) An artificially-created structure in which a tiny 'speck' of one *semiconducting material* is entirely embedded in a larger sample of another semiconducting material. The embedded speck is typically a few nanometres across. Such a structure can be modelled as a microscopic three-dimensional box in which *electrons* can be confined.

quantum mechanics (B1: 7) The comprehensive quantum theory of *systems* of finite numbers of *particles* that superseded *old quantum theory* and *classical mechanics*. Quantum mechanics has both non-relativistic and relativistic branches, but it does not cover systems in which particles are created

or destroyed: that is the province of quantum field theory. This course focuses mainly on the non-relativistic aspects of quantum mechanics.

quantum number (B1: 57) A discrete index (often, but not always, an integer) used to label discrete *eigenvalues*, *eigenfunctions*, *stationary state wave functions* or *quantum states* in a given *system*. For example, a *particle* in a *one-dimensional infinite square well* has *energy eigenvalues* labelled by a positive integer n..

A single quantum number is enough to specify the quantum state of a spinless particle in a one-dimension. At least three quantum numbers are needed to fully specify a state in a three-dimensional system.

quantum physics (B1: 7) A term given to any branch of physics that is based on quantum ideas. For example, aspects of nuclear physics, atomic physics or quantum field theory may be classified as being quantum physics.

quantum random number generator (B1: 15) A device that uses the fundamental *probabilistic* nature of *quantum mechanics* to generate a sequence of *random* numbers.

quantum state A specified *state* of a *quantum system*. *Quantum mechanics* is *indeterministic*, so precise knowledge of the quantum state of a system may not allow us to predict with certainty the results of all *measurements* taken on the system.

quantum system A *system* that is analyzed according to the principles of *quantum physics*, rather than *classical physics*. Since quantum physics is assumed to be universally valid, any system might be taken to be a quantum system; in practice, macroscopic systems are often taken treated classically without discernible error.

quantum wafer (B1: 64) An artificially-created structure in which a thin layer of one *semiconducting material* is sandwiched between thicker layers of another semiconducting material. The thin layer is typically a few nanometres thick. Such a structure allows *electrons* to move freely in two dimensions while being narrowly confined in the third.

quantum wire (B1: 63) An artificially-created structure in which a thin 'thread' of one *semiconducting material* is embedded in another semiconducting material. The embedded thread is typically a few nanometres across. Such a structure allows *electrons* to move freely in one dimension while being narrowly confined in the other two dimensions.

radiative transition (B1: 145) A process in which a *system* (such as an *atom*) undergoes a transition from one *quantum state* to another by the emission or absorption of *electromagnetic radiation*. Such a process involves the creation or destruction of a *photon* (or, very rarely, more than one photon).

radioactive decay A process in which an unstable *nucleus* loses *energy* by emitting ionizing *particles* and *electromagnetic radiation* and, as a result, transforms into a different type of nucleus.

raising operator (B1: 137) For a *harmonic oscillator* with *length parameter* a, the raising operator is defined as

$$\widehat{\mathrm{A}}^\dagger = \frac{1}{\sqrt{2}}\Big(\frac{x}{a} - a\frac{\partial}{\partial x}\Big),$$

This *operator* converts an *energy eigenfunction* $\psi_n(x)$ of the oscillator into the next eigenfunction of higher *energy*. If the eigenfunctions are *normalized*,

$$\widehat{\mathrm{A}}^\dagger\psi_n(x) = \sqrt{n+1}\,\psi_{n+1}(x).$$

for $n = 0, 1, 2, \ldots$. See also *lowering operator* and *ladder operator*.

Ramsauer–Townsend effect (B1: 196) A sharp dip in the measured *total cross-section* for the *scattering* of *electrons* by noble gas *atoms* (such as xenon) at an *energy* of about 1 eV. This is a three-dimensional analogue of the *transmission resonance* found in one dimension.

random (B1: 227) A variable is said to be random if its possible values have definite *probabilities*, but no further information is available to us about which of its values will be obtained.

real axis (B1: 212) An axis in the *complex plane* on which *complex numbers* have zero *imaginary part* and which points in the direction of increasing *real part*.

real number An ordinary number; in other words, a *complex number* with no *imaginary part*.

real part (B1: 210) Given a *complex number* $z = x + \mathrm{i}y$, where x and y are *real numbers*, the real part of z is equal to x. The real part of any complex expression is given by

$$\mathrm{Re}(z) = \frac{z + z^*}{2},$$

where z^* is the *complex conjugate* of z.

reduced mass (B1: 128) A mass used to characterize a two-particle *system*. If the two *particles* have masses m_1 and m_2, the reduced mass of the two-particle system is

$$\mu = \frac{m_1 m_2}{m_1 + m_2}.$$

For two particles with positions $\mathbf{r}_1$ and $\mathbf{r}_2$, interacting via a mutual *potential energy function* $V(\mathbf{r}_2 - \mathbf{r}_1)$, the relative position $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$ obeys the same equations as a single particle of mass μ in a potential energy well $V(\mathbf{r})$.

The reduced mass is always smaller than the mass of either of the particles in the system. In a *hydrogen*

atom, μ is 0.9995 the mass of an *electron*, but for a *diatomic molecule* composed of two similar *atoms*, it is half the mass of either atom.

reflection coefficient (B1: 182) The *probability* that a specified one-dimensional *potential energy function* will cause an incident *particle* to reverse its direction of motion. Compare with *transmission coefficient*.

relative frequency (B1: 229) If a quantity A is measured N times and the result A_i is obtained on N_i occasions, the relative frequency of the result A_i is N_i/N. In the long run, this relative frequency is expected to tend to the corresponding *probability*, p_i.

restoring force (B1: 125) A force that acts in a direction which tends to restore a *particle* towards its equilibrium position.

sandwich integral (B1: 112) An integral of the form

$$\int_{-\infty}^{\infty} \Psi^*(x,t)\,\widehat{\mathrm{A}}\,\Psi(x,t)\,\mathrm{d}x,$$

used in the *sandwich integral rule*.

sandwich integral rule (B1: 112) In a *quantum state* described by the *wave function* $\Psi(x,t)$, the *expectation value* of an *observable* A is given by the *sandwich integral*

$$\langle A\rangle = \int_{-\infty}^{\infty} \Psi^*(x,t)\,\widehat{\mathrm{A}}\,\Psi(x,t)\,\mathrm{d}x,$$

where $\widehat{\mathrm{A}}$ is the quantum-mechanical *operator* corresponding to A.

scalar A quantity that is completely specified by a single number, or by a number times an appropriate unit of measurement. Contrast with *vector*.

scanning tunnelling microscope (B1: 206) A type of microscope that produces atomic-scale maps of surface structure by monitoring the *tunnelling* of *electrons* through the tiny gap between a sample's surface and a very narrow probe tip that is scanned across the surface.

scattering (B1: 178) A process in which an incident *particle* or *wave* is affected by interaction with some kind of target, quite possibly another particle. The interaction can affect the incident particle in a number of ways; it may change its *kinetic energy*, direction of motion or *quantum state* of internal excitation. Particles can even be created, destroyed or absorbed. Scattering may be *elastic* or *inelastic*.

In one dimension, scattering can be described in terms of the *reflection coefficient* and the *transmission coefficient*. More generally, it is described in terms of the *total cross-section* and the *differential cross-section*.

Schrödinger's equation (B1: 24, 46) The *partial differential equation* that governs the time development of the *wave function* Ψ describing the *state* of a *system* in *wave mechanics*. Schrödinger's equation can be written in the general form

$$\mathrm{i}\hbar\frac{\partial\Psi}{\partial t} = \widehat{\mathrm{H}}\Psi,$$

where $\widehat{\mathrm{H}}$ is the *Hamiltonian operator* of the system and $\hbar$ is *Planck's constant* divided by 2π.

In any specific situation $\widehat{\mathrm{H}}$ takes a form that characterizes the system under consideration, and Ψ depends on the possible coordinates of *particles* in the system as well as on time. For example, in the case of a one-dimensional system consisting of a particle with *potential energy function* $V(x)$, Schrödinger's equation takes the form

$$\mathrm{i}\hbar\frac{\partial\Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x)\Psi(x,t).$$

second-order partial derivative (B1: 224) A function obtained by partially differentiating another function twice (possibly with respect to different *independent variables*).

selection rules (B1: 131) Rules that govern whether particular *radiative transitions* are allowed or (to a first approximation at least) forbidden. In a *harmonic oscillator*, for example, a selection rule restricts radiative transitions to those between neighbouring *energy levels*.

semiconducting material (B1: 63) A material with an electrical conductivity intermediate between those of conductors and insulators. Examples include silicon and germanium.

separable partial differential equation (B1: 226) A *partial differential equation* for which the method of *separation of variables* can be applied successfully. Special solutions of the partial differential equation can then be found which are products of single-variable functions of all the *independent variables* in the equation.

separation constant (B1: 54, 226) An undetermined constant that appears when a *partial differential equation* is solved by the method of *separation of variables*. The separation constant then appears in the *eigenvalue equations* for the separated variables.

separation of variables (B1: 53, 225) A method used for finding some solutions of some *partial differential equations*. It involves looking for solutions that are products $X(x)Y(y)\ldots$ of single-variable functions of each *independent variable* in the equation. Substituting a solution of this form into the partial differential equation and rearranging terms leads to an equation in which one variable appears only on the left-hand side and the others appear only on the right. Each side of the equation can then be equated to the same *separation constant*. This procedure is repeated for the remaining variables until

each of the functions $X(x)$, $Y(y)$ appears in its own *ordinary differential equation*. Multiplying the solutions of these ordinary differential equations together gives special solutions of the original partial differential equation.

In the case of *Schrödinger's equation* for a one-dimensional *system* consisting of a *particle* with a given *potential energy function* $V(x)$ that is independent of time, a product solution takes the form $\psi(x)T(t)$, where $\psi(x)$ is a solution of the *time-independent Schrödinger equation* for the system, $T(t) = \mathrm{e}^{-\mathrm{i}Et/\hbar}$, and the separation constant E represents the total *energy* of the system.

SI An internationally agreed system of units of measurement. The system employs seven base units, including the kilogram (abbreviated to kg), the metre (abbreviated to m), the second (abbreviated to s), the ampere (abbreviated to A) and the *kelvin* (abbreviated to K). It also includes a number of derived units obtained by combining base units in various ways.

The SI system uses certain standard prefixes such as kilo $= 10^3$, mega $= 10^6$, giga $= 10^9$, tera $= 10^{12}$, milli $= 10^{-3}$, micro $= 10^{-6}$, nano $= 10^{-9}$ and pico $= 10^{-12}$. It also recognizes a number of standard symbols and abbreviations. SI itself is one of these symbols and stands for Système International.

simple harmonic motion (B1: 124) In *classical physics*, a particular type of *oscillation* of a *particle* about a specified equilibrium position, characterized by the fact that the acceleration of the particle is always directed towards the equilibrium position and is proportional to the displacement from that point. In one dimension, simple harmonic motion may be described by a *differential equation* of the form

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + \omega_0^2 x = 0,$$

which has the *general solution*

$$x(t) = A\cos(\omega_0 t + \phi),$$

where A is the *amplitude* of the motion, ω_0 is the *angular frequency* and ϕ is the *phase constant* of the motion.

simple harmonic oscillation Another term for *simple harmonic motion*.

simple harmonic oscillator In *classical physics*, a *particle* or *system* that performs *simple harmonic motion*. Another term for a *harmonic oscillator*.

sinusoidal Any function of the form

$$f(x) = A\sin(kx + \phi)$$

where A, k and ϕ are *real* constants, may be described as a sinusoidal function. Thus any *linear combination* of $\sin x$ and $\cos x$, with real coefficients, is a sinusoidal function.

spectra The plural of *spectrum*.

spectral lines (B1: 9) Narrow lines (corresponding to narrow ranges of *frequency* or *wavelength*) seen in the *spectra* of substances and characteristic of those substances. Each spectral line results from a *radiative transition* and has a frequency $f = \Delta E/h$, where ΔE is the *magnitude* of the *energy* difference between the initial and final *quantum states* and h is *Planck's constant*.

spectrum (B1: 9) Any particular distribution of *electromagnetic radiation*, expressed as a function of *intensity* versus *wavelength*, *frequency* or a related quantity such as *photon energy*. Many spectra consist of *spectral lines*, produced during *radiative transitions* between discrete *quantum states*. Such a spectrum can provide an identifiable 'fingerprint' of a substance. .

speed of light The speed of an electromagnetic wave. In an *inertial frame of reference*, in a vacuum, this is a universal constant $c = 3.00 \times 10^8\,\mathrm{m\,s^{-1}}$, independent of *amplitude*, *frequency*, *wavelength* or motion of the source.

standard deviation (B1: 116, 230) A quantity that measures the extent by which a set of data values spreads out on either side of the *average value*. The standard deviation of a quantity A is defined as

$$\sigma(A) = \left[\,\overline{\left(A - \overline{A}\right)^2}\,\right]^{1/2},$$

where the bars indicate average values. As the number of data values tends to infinity, the standard deviation $\sigma(A)$ is expected to approach the *uncertainty* ΔA.

(Statisticians sometimes use a slightly different quantity called the sample standard deviation. This differs from our definition by an amount that becomes vanishingly small as the number of data values tends to infinity.)

standing wave (B1: 73) A *wave* that oscillates without travelling through space. All points in the disturbance that constitutes the wave oscillate *in phase* with the same *frequency* but with different *amplitudes*. The fixed points of zero disturbance are called the *nodes* of the wave (although end-points of the disturbance are not always counted as nodes). A standing wave can be regarded as the sum of two *travelling waves*, propagating in opposite directions.

A familiar example from *classical physics* is the standing wave on a string stretched between fixed endpoints at $x = 0$ and $x = L$. This can be represented by the function

$$u(x,t) = A\sin(kx)\cos(\omega t + \phi),$$

where ω is the *angular frequency* and k is the *wave number*. The possible standing waves are restricted by the requirement that the distance between the fixed ends of the string must be equal to a whole number of half-*wavelengths*, which implies that $kL = n\pi$, where n is an integer.

In *quantum physics*, a *stationary-state wave function* describes a *complex*-valued standing wave.

state (B1: 51) The condition of a *system* described in sufficient detail to distinguish it from other conditions in which the system would behave differently. In *classical mechanics*, the state of a system of *particles* at a given time can be completely specified by giving the values the positions and velocities of all the particles at that time. In *wave mechanics*, the state of a system at a given time is specified by the *wave function* at that time. Everything that can be said about the *probabilities* of the outcomes of *measurements* is implicit in the wave function.

stationary state (B1: 57) A *quantum state* described by a *wave function* that satisfies *Schrödinger's equation* and is a product of a factor that depends on spatial coordinates and a factor that depends on time. In one dimension, the wave function of a stationary state can be expressed as $\psi(x)\,\mathrm{e}^{-\mathrm{i}Et/\hbar}$, where E is the total *energy* of the *system*, $\psi(x)$ is a solution of the *time-independent Schrödinger equation* corresponding to the *energy eigenvalue* E, and $\hbar$ is *Planck's constant* divided by 2π.

In any stationary state, the energy has a definite value and the *probability distribution* associated with any *observable* is independent of time.

STM An acronym for *scanning tunnelling microscope*.

superposition principle (B1: 52) See *principle of superposition*.

symmetric well (B1: 76) In one dimension, a well described by a *potential energy function* $V(x)$ with the property $V(x) = V(-x)$. Symmetric wells have *energy eigenfunctions* that are either *even functions* or *odd functions*. The even or odd nature of eigenfunctions alternates with increasing *energy*.

system The portion of the Universe chosen as the subject of a scientific investigation. See also *isolated system*.

three-dimensional infinite square well (B1: 82) A three-dimensional *potential energy function* that is infinite everywhere except for a finite region, within which the *potential energy* has a constant value (usually taken to be zero). In a three-dimensional infinite square well occupying a cubic region with sides of length L, with zero potential energy within the well, a *particle* of mass m has *energy eigenvalues*

$$E_n = \frac{(n_x^2 + n_y^2 + n_z^2)\hbar^2}{2mL^2},$$

where the *quantum numbers* n_x, n_y and n_z are positive integers. The *energy eigenfunctions* are products of a *sinusoidal* function in x, a sinusoidal function in y and a sinusoidal function in z. Most of these eigenfunctions are *degenerate*.

time-independent Schrödinger equation (B1: 55) An *eigenvalue equation* for *energy*, which can be derived from *Schrödinger's equation* for a *system* of *particles* with a time-independent *potential energy function*. The time-independent Schrödinger equation can be written in the form

$$\widehat{\mathrm{H}}\psi = E\psi,$$

where $\widehat{\mathrm{H}}$ is the *Hamiltonian operator* of the system, ψ is an *energy eigenfunction* and E is the corresponding *energy eigenvalue*.

For a one-dimensional system consisting of a particle with potential energy function $V(x)$, the time-independent Schrödinger equation is

$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\psi(x)}{\mathrm{d}x^2} + V(x)\psi(x) = E\psi(x),$$

where $\hbar$ is *Planck's constant* divided by 2π.

total cross-section (B1: 196) A quantity used to measure the total rate per unit time per unit incident *flux*, at which a given type of target scatters a given type of incident *particle*. The *SI* unit of total cross-section is m^2 but the *barn* is a more commonly-used unit in particle and nuclear physics.

transmission coefficient (B1: 182) The *probability* that a specified one-dimensional *potential energy function* will allow an incident *particle* to pass through, continuing in its original direction of motion. Compare with *reflection coefficient*.

transmission resonance (B1: 194) For one-dimensional *scattering*, a transmission resonance is a maximum in the *transmission coefficient* T as a function of *energy*, ideally with $T = 1$, corresponding to perfect transmission and no reflection.

travelling wave A *wave* that propagates from one place to another. A one-dimensional example of such a wave is represented by the function

$$u(x,t) = A\cos(kx - \omega t + \phi),$$

where the positive constant A is the *amplitude*, k is the *wave number*, ω is the *angular frequency* and ϕ is the *phase constant*. The quantity $kx - \omega t + \phi$ is the *phase* of the wave. Contrast with *standing wave*.

tunnelling (B1: 179) The quantum-mechanical phenomenon in which an incident *particle*, initially in a classically-allowed region, is able to pass through a *classically-forbidden region* and emerge on the far side of it, travelling in another classically-allowed region. Such tunnelling cannot occur in *classical physics* because the particle is unable to enter the classically-forbidden region, where the particle's total *energy* is less than its *potential energy*.

Tunnelling occurs in *alpha decay*, *nuclear fusion* in the *proton–proton chain* and the *scanning tunnelling microscope*.

two-dimensional infinite square well (B1: 78) A two-dimensional *potential energy function* that is infinite everywhere except for a finite region, within which the *potential energy* has a constant value (usually taken to be zero). In a two-dimensional infinite square well occupying a square region with sides of length L, with zero potential energy within the well, a *particle* of mass m has *energy eigenvalues*

$$E_n = \frac{(n_x^2 + n_y^2)\hbar^2}{2mL^2},$$

where the *quantum numbers* n_x and n_y are positive integers. The *energy eigenfunctions* are products of a *sinusoidal* function in x and a sinusoidal function in y. Most of these eigenfunctions are *degenerate*.

two-slit interference pattern (B1: 18) The pattern of *interference maxima* and *interference minima* that forms when a *plane wave* is incident on an absorbing screen containing two narrow slits. The pattern, formed on the far side of the absorbing screen, consists of a series of bright bands separated by dark bands. The bright bands occur in places where the waves passing through the two slits interfere constructively. The dark bands occur in places where these waves interfere destructively. The two slits must be narrow enough for their two *diffraction patterns* to appreciably overlap. This implies that their widths must be comparable to, or smaller than, the *wavelength* of the incident wave. See also *constructive interference* and *destructive interference*.

uncertainty (B1: 116, 231) The quantum-mechanical prediction for the *standard deviation* of an *observable* in a *system* in a given *state*. The uncertainty is defined by

$$\Delta A = \left[\left\langle (A - \langle A \rangle)^2 \right\rangle\right]^{1/2}$$

where angular brackets denote *expectation values* and $(A - \langle A \rangle)^2$ is the square of the deviation of A from its expectation value. Uncertainties can also be calculated from the formula

$$\Delta A = \left[\langle A^2 \rangle - \langle A \rangle^2\right]^{1/2}.$$

vector A quantity with a definite *magnitude* and a definite direction in space. Vectors can be specified by giving their components in a given coordinate system.

visible light *Electromagnetic radiation* with *wavelength* between about 4×10^{-7} m (400 nm, violet) and 7×10^{-7} m (700 nm, red), or equivalently with *frequency* between about 8×10^{14} Hz and 4×10^{14} Hz.

watt The *SI* unit of power, or of the rate at which *energy* is transferred, represented by the symbol W. One watt is defined by $1\,\mathrm{W} = 1\,\mathrm{J\,s^{-1}}$.

wave A periodic disturbance that travels from one point to another. If the wave travels through a material medium, no *particle* in the medium is permanently displaced by passage of the wave. Waves may be *standing* or *travelling*. Some waves may be *transverse* or *longitudinal*. Transverse waves may be characterized by their polarization. Waves may also be characterized by their *frequency*, *wavelength* and direction of propagation.

wave function (B1: 24, 49) A function that fully describes the *quantum state* of a *system* in *wave mechanics*. For a single *particle* in one dimension, the wave function takes the form $\Psi(x,t)$. The wave function evolves in time according to *Schrödinger's equation*, except when the system is disturbed by a *measurement*, leading to the *collapse of the wave function*.

The spatial extent of a particle's wave function is not to be confused with the size of the particle. For example, the spatial extent of the wave function of an *electron* in a *hydrogen atom* is the spatial extent of the atom, whilst the electron itself is much smaller, a point particle as far as we can tell.

wave mechanics (B1: 7) A term given to a way of formulating *quantum mechanics*, and carrying out quantum-mechanical calculations, pioneered by Erwin Schrödinger. In wave mechanics, the *state* of a *system* is described by a *wave function*, which obeys a *partial differential equation* called *Schrödinger's equation*. *Observable* quantities are represented by *linear differential operators* that act on the wave function.

wave number (B1: 17) A quantity $k = 2\pi/\lambda$ that describes a *monochromatic wave*, where λ is the *wavelength* of the wave. Wave numbers label *momentum eigenfunctions* in one dimension.

wave packet (B1: 150) A *wave function* that is a discrete or continuous *linear combination* of two or more different *stationary-state* wave functions. Any wave function (other than one describing a stationary state) can normally be expressed in this way.

wave-packet spreading (B1: 172) The phenomenon whereby a *particle* described by a *wave packet* has an *uncertainty* in position that changes (normally increases) with time.

wave–particle duality (B1: 17) The phenomenon whereby *systems* display properties associated with both *particles* and *waves* according to the kind of *measurements* performed on them. It shows that it is best to think of 'particle' and 'wave' as categories associated with our macroscopic world; there is no obligation for microscopic entities to fall wholly into one category or the other.

wavelength (B1: 17) The spatial separation λ, measured along the direction of propagation, of successive points in a *wave* that differ in *phase* by 2π at any fixed time t. More crudely, the distance between successive peaks of the wave.

zero of potential energy A point at which the *potential energy* of a *particle* is set equal to zero.

zero-point energy (B1: 130) The *ground-state energy* of a *particle*, measured from the bottom of its *potential energy* well. This energy is present even at the *absolute zero* of temperature.

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